

Viscous Evolution of a Quark Gluon Plasma and Radiative E-Loss

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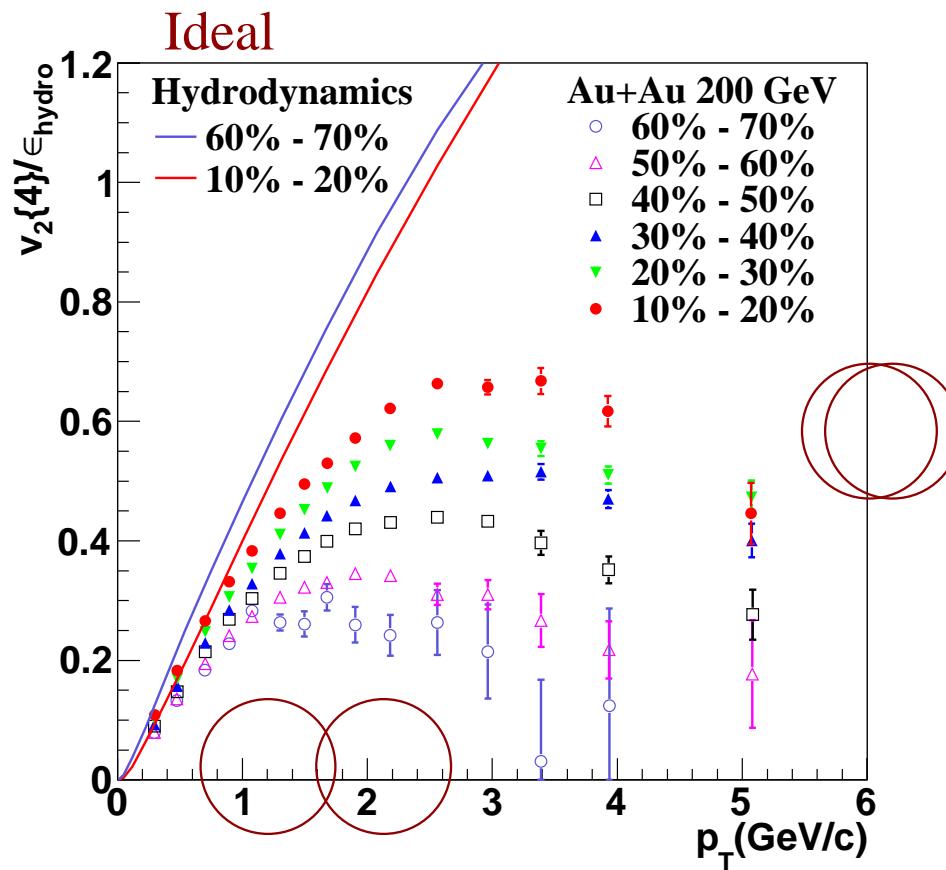
Kevin Dusling, Guy Moore, DT, arXiv:0909.0754



Teaney

v_2 as a function of transverse momentum and p_T :

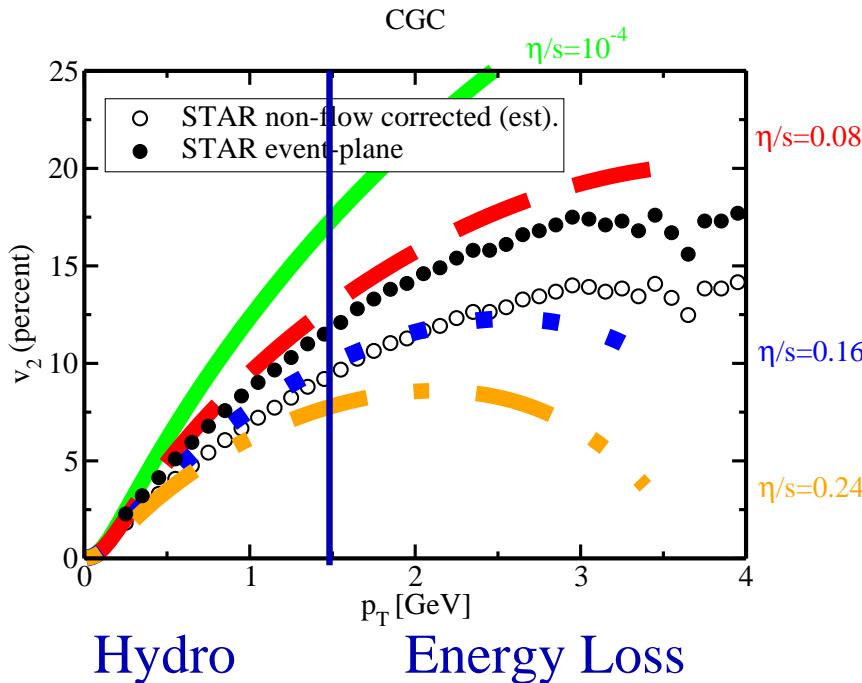
$$\frac{v_2}{\epsilon} = \frac{\text{Flow Response}}{\text{External Perturbation}}$$



Systematic trends versus p_T indicate viscosity and hydro

Three Points:

(Romatschke Luzum)



1. $v_2(p_T)$ for $p_T \lesssim 1.0$ GeV is universal
 - Depends only on η/s
2. High momentum *thermalized particles* are not universal
 - How energy loss and \hat{q} influences hydro v_2 predictions.
3. Explain “quark number scaling”:
 - Without coalescence

Viscous Corrections

1. Viscous corrections to the equation of motion

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{with} \quad T^{\mu\nu} = \underbrace{(e + p)u^\mu u^\nu + pg^{\mu\nu}}_{\text{ideal}} - \underbrace{2\eta \langle \partial^\mu u^\nu \rangle}_{\text{viscous } \pi^{\mu\nu}}$$

2. Viscous corrections to the distribution function

$$f \rightarrow f_o + \delta f$$

- General form in rest frame and ansatz

$$\delta f = -\chi(p) \times f_o(p) \hat{p}^i \hat{p}^j \langle \partial_i u_j \rangle$$

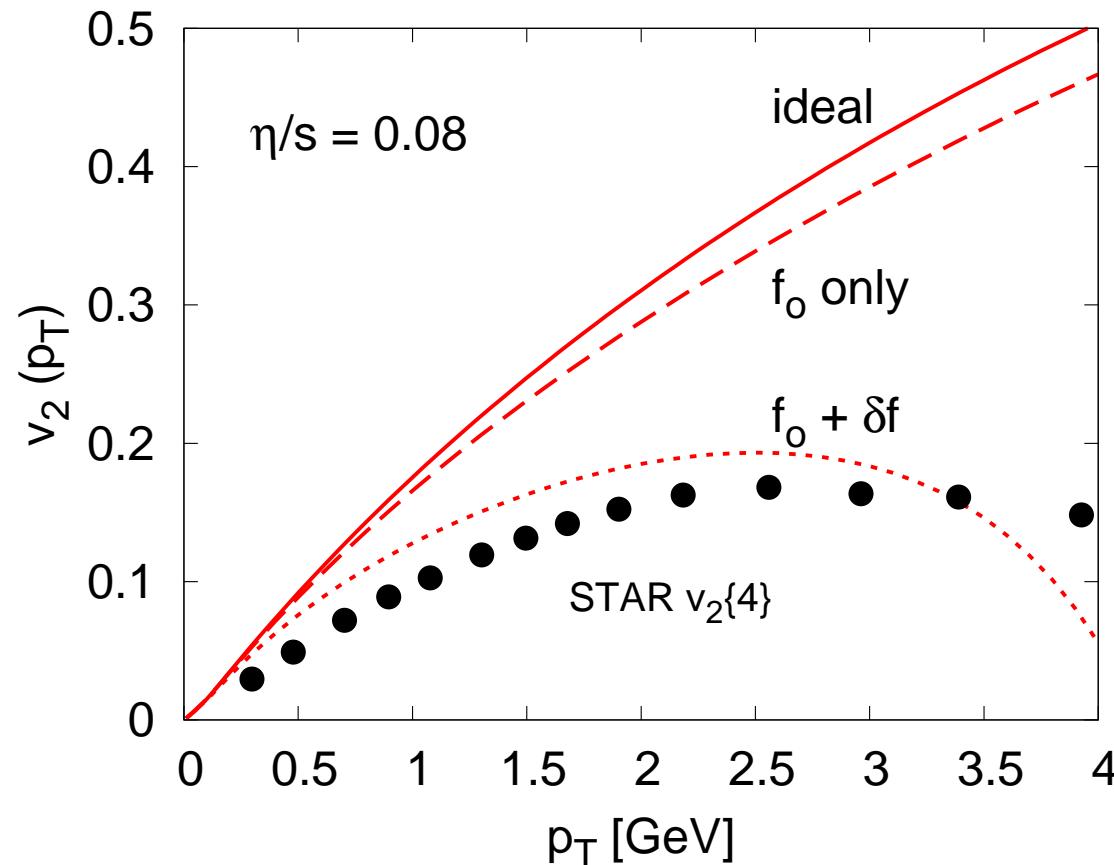
- The Quadratic Ansatz $\chi(p) \propto p^2$

$$\delta f = C p^2 \times f_o(p) \hat{p}^i \hat{p}^j \langle \partial_i u_j \rangle$$

All simulations have used the quadratic ansatz!

The role of δf

Pure glue, $e_{\text{frz}} = 0.6 \text{ GeV/fm}^3$



We should understand δf and the Quadratic Ansatz!

Calculating δf : Relaxation Time Approximation

$$\left[\partial_t + v_{\mathbf{p}} \frac{\partial}{\partial x} \right] f = - \frac{\delta f}{\tau_R(p)}$$

1. Substitute $f = n_p + \delta f$ with

$$\left[\partial_t + v_{\mathbf{p}} \frac{\partial}{\partial x} \right] n_p = - \frac{\delta f}{\tau_R(E_{\mathbf{p}})} \quad \text{with} \quad n_p = \frac{1}{e^{-P \cdot u(\mathbf{x}, t)} \mp 1}$$

2. With algebra and massless classical statistics (here):

$$\delta f = -\chi(p) \times n_{\mathbf{p}} \hat{p}^i \hat{p}^j \langle \partial_i u_j \rangle \quad \text{with} \quad \chi(p) = \tau_R(p) \frac{p}{T}$$

Quadratic ansatz corresponds to $\tau_R \propto p$.

What about $\tau_R \propto p^\beta$?

Two Extreme Limits: Quadratic and Linear Ansatz

$$\delta f = - \underbrace{\frac{\tau_R(p)}{Tp}}_{\equiv \chi(p)} \times n_p p^i p^j \langle \partial_i u_j \rangle$$

For the relaxation time take

$$\tau_R(p) \sim \frac{p}{\frac{dp}{dt}}$$

1. Relaxation time growing with parton energy – “collisional e-loss”

$$\tau_R \propto p \quad \frac{dp}{dt} \propto \text{const} \quad \chi(p) \propto p^2$$

2. Relaxation time independent of parton energy – “extreme rad. e-loss”

$$\tau_R \propto \text{Const} \quad \frac{dp}{dt} \propto p \quad \chi(p) \propto p$$

Reality is probably in-between

δf is constrained by shear viscosity

$$T^{ij} = p\delta^{ij} - \eta \langle \partial^i u^j \rangle = \int_{\mathbf{p}} \frac{p^i p^j}{E} (n_p + \delta f)$$

- So, if we write our ansatz

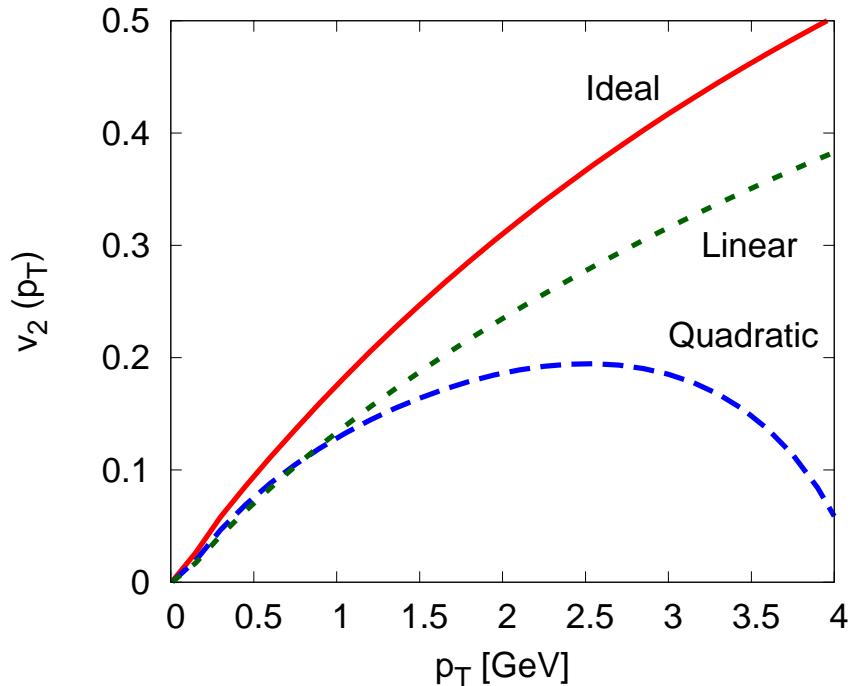
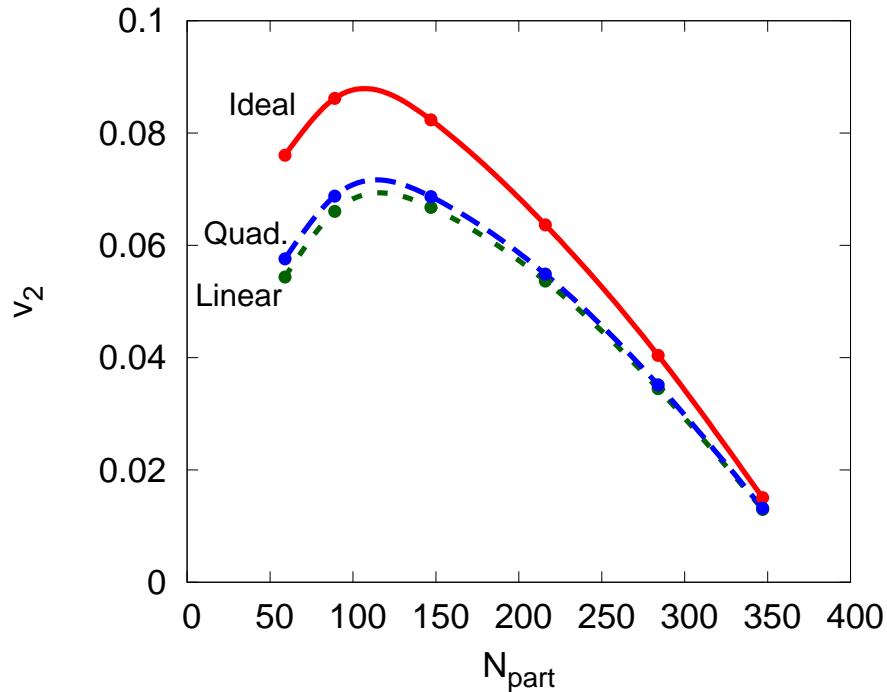
$$\delta f = -\chi(p) \times n_p \hat{p}^i \hat{p}^j \langle \partial_i u_j \rangle$$

Find the first moment of δf determines the shear viscosity

$$\eta = \frac{1}{15} \int \frac{d^3 p}{(2\pi)^3} n_p \chi(p) p$$

Ansätze partially constrained by shear viscosity

Two Limits: Quadratic and Linear Ansatz – Same η/s



1. The mean \bar{v}_2 is universal depending on η/s and not micro details
2. Higher momentum depends strongly on micro-details of energy loss

What is reality? Quadratic or Linear?

Lets calculate δf in pQCD!

Solving for δf with the Boltzmann Equation:

$$\partial_t f + \mathbf{v}_p \cdot \partial_x f = C \circ f$$

- Substitute $f = n_p + \delta f$

$$\partial_t n_p + \mathbf{v}_p \cdot \partial_x n_p = \underbrace{C \circ \delta f}_{\equiv C_{pp'} \delta f_{p'}}$$

- with a bit of algebra

$$n_p \frac{p^i p^j}{T E_p} \langle \partial_i u_j \rangle = C \circ \delta f$$

- The collisions and bremsstrahlung is all in C .

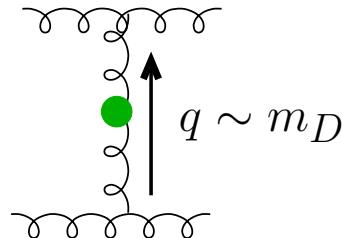


Can go invert this matrix and determine δf

Three Models of Energy Loss

1. Soft Scattering

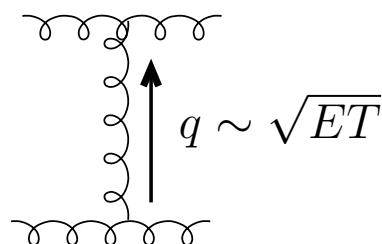
$$\frac{dp}{dt} \propto C_R \alpha_s^2 T^2 \log \left(\frac{T}{m_D} \right)$$



find $\chi(p) \propto p^2$

2. Collisional Energy Loss

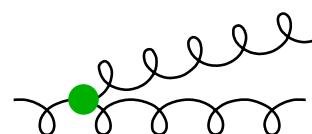
$$\frac{dp}{dt} \sim C_R \alpha_s^2 T^2 \log \left(\frac{p}{m_D} \right)$$



find $\chi(p) \propto \frac{p^2}{\log(p\mu)}$

3. Radiative + Collisional Energy Loss in Infinite medium

$$\frac{\Delta p}{\Delta t} \sim \alpha_s \sqrt{\hat{q} E_p}$$

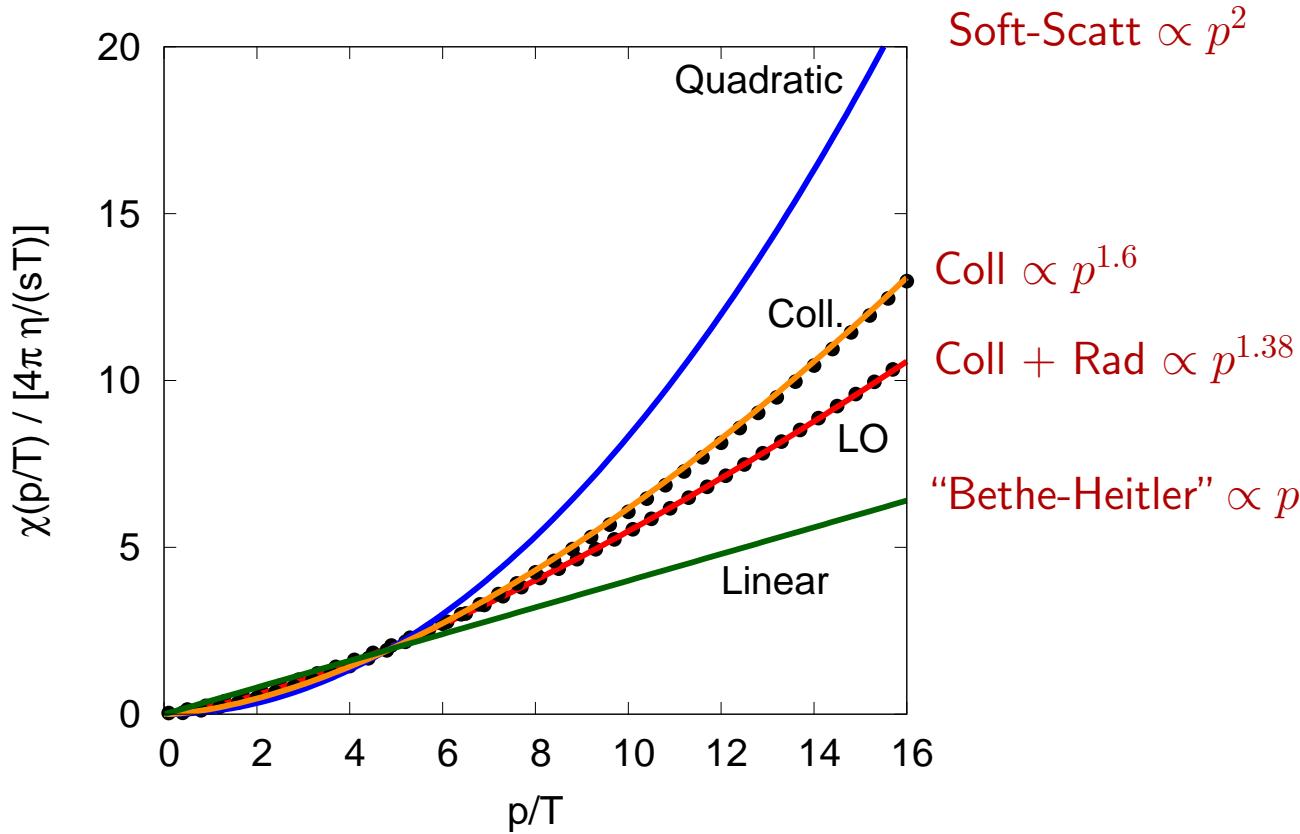


find $\chi \propto p^{3/2}$

These estimates are borne out by our numerical work

Summary – Energy Loss and δf

$$\delta f = -\chi(p) n_p \hat{p}^i \hat{p}^j \langle \partial_i u_j \rangle \quad \text{fit with} \quad C p^{2-\alpha}$$



Soft-Scatt $\propto p^2$

Coll $\propto p^{1.6}$

Coll + Rad $\propto p^{1.38}$

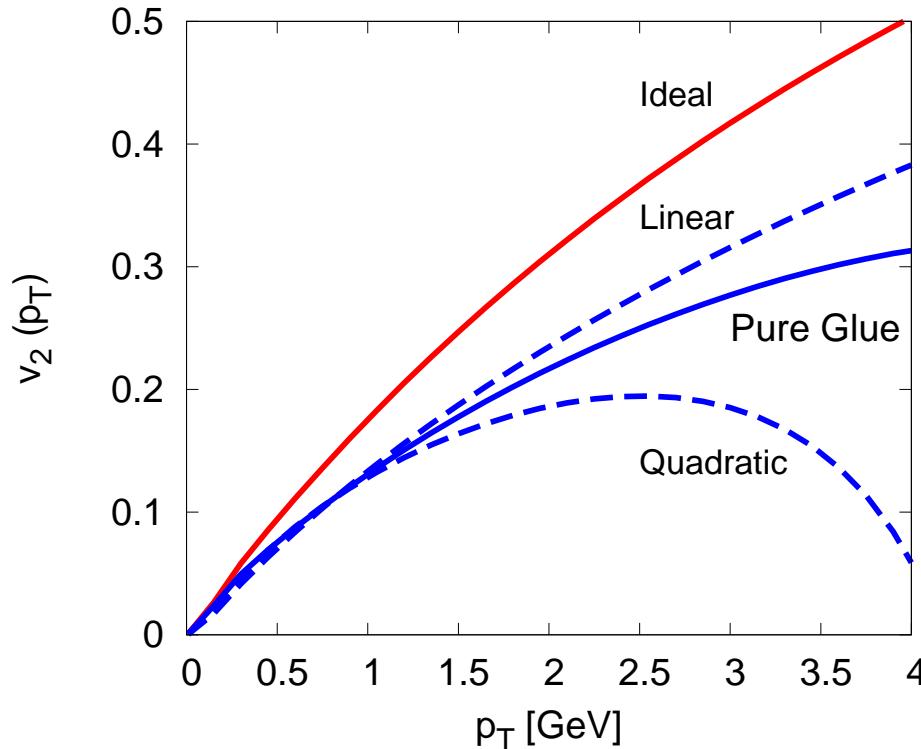
"Bethe-Heitler" $\propto p$

Energy loss determines $\chi(p)$

QCD kinetic theory expectation $\chi(p) \propto p^{1.38}$ in relevant range

Phenomenological Summary

pure glue, $\eta / s = 0.08$, $e_{\text{frz}} = 0.6 \text{ GeV/fm}^3$



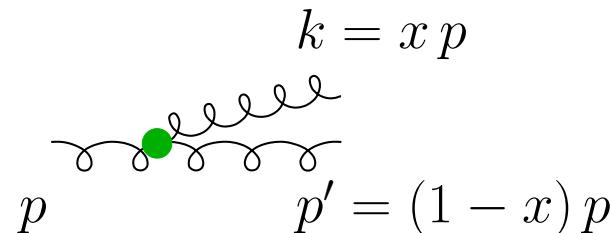
pQCD is closer to a linear ($\tau_R = \text{const}$) rather than a quadratic ansatz

Connection to energy radiative loss – high momentum but thermal

- At large momentum brem dominates the Boltzmann collision term

$$\partial_t f + \mathbf{v}_p \cdot \partial_{\mathbf{x}} f = -\mathcal{C}^{1 \leftrightarrow 2}[f].$$

- The collision kernel is



$$\mathcal{C}^{1 \rightarrow 2} \propto \underbrace{\int_0^1 dx}_{\text{Phase Space}} \times \underbrace{\gamma_{gg}^g(p; p', k)}_{|\mathcal{M}|^2} \times \underbrace{[f_p(1 + f_{p'})(1 + f_k)]}_{\text{Stimulation Factors}}$$

- The QCD splitting function is medium modified

see P.Arnold, C.Dogan, BDMPS

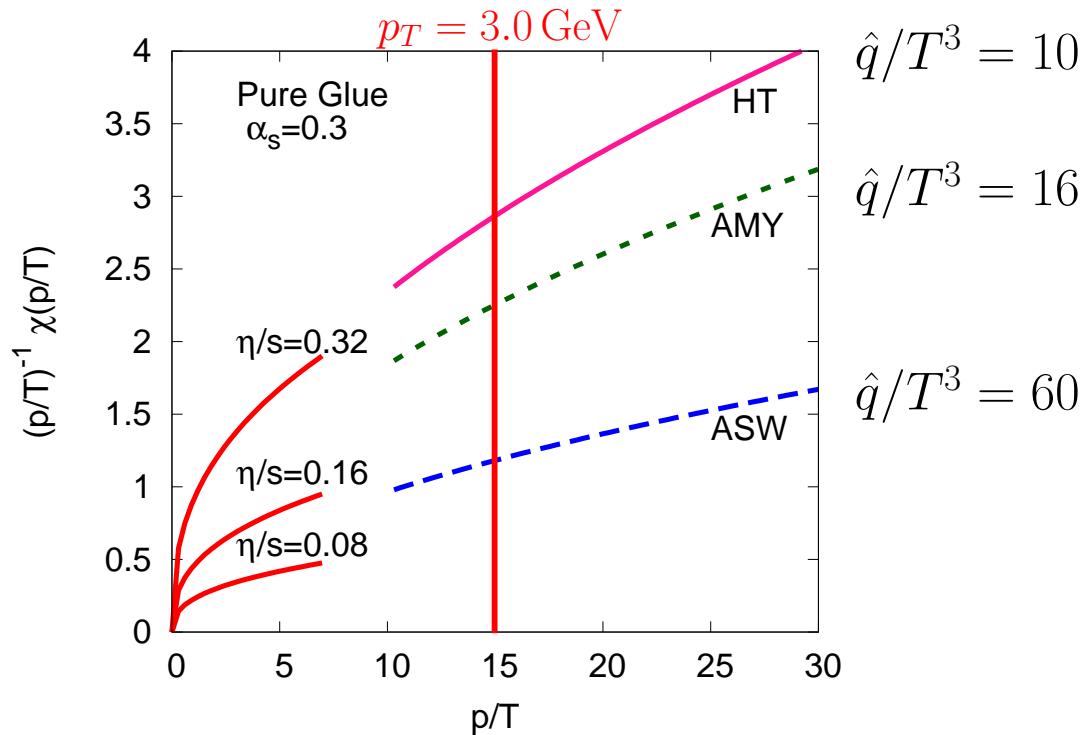
$$\gamma_{gg}^g \propto \alpha_s C_A d_A \sqrt{p \hat{q}} \frac{[1 - x(1 - x)]^{5/2}}{[x(1 - x)]^{3/2}}$$

\hat{q} and viscous corrections at high momenta: A nifty formula

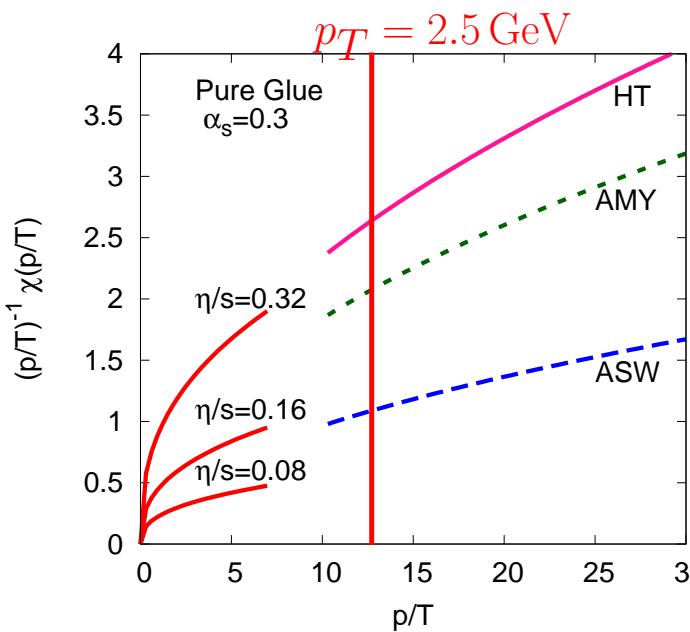
$$\chi(p) = 0.7 \times \underbrace{\frac{p^{3/2}}{\alpha_s T \sqrt{\hat{q}}}}_{\text{radiative loss}}$$

Viscous Correction

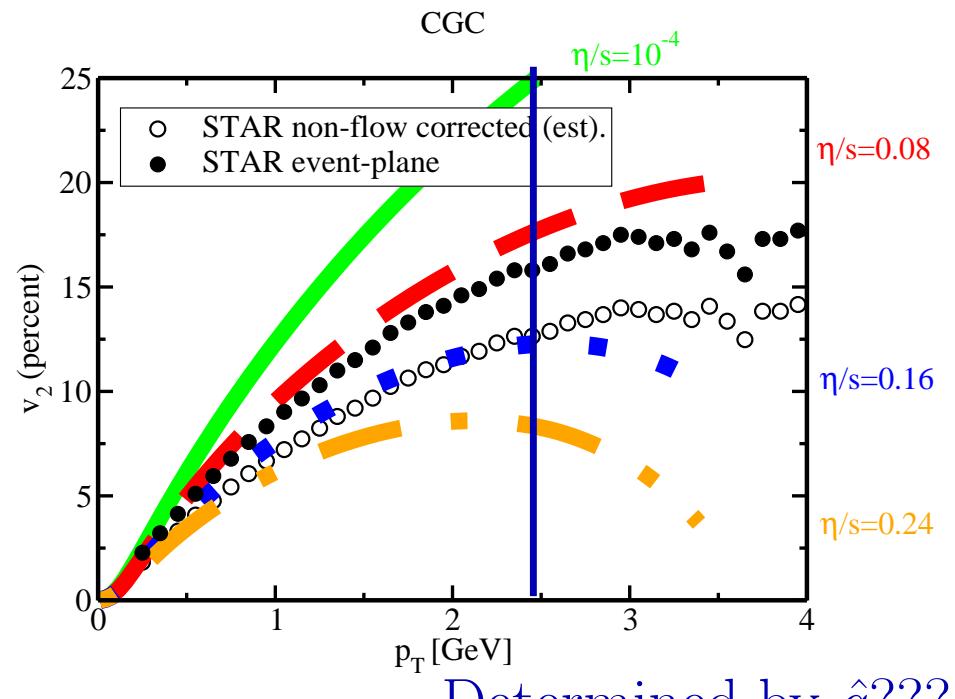
1. At low momentum $\chi(p)$ is determined by the shear viscosity η/s
2. At high momentum $\chi(p)$ is determined by \hat{q}



How much of $v_2(p_T)$ at $2 - 3$ GeV is due to \hat{q} ?

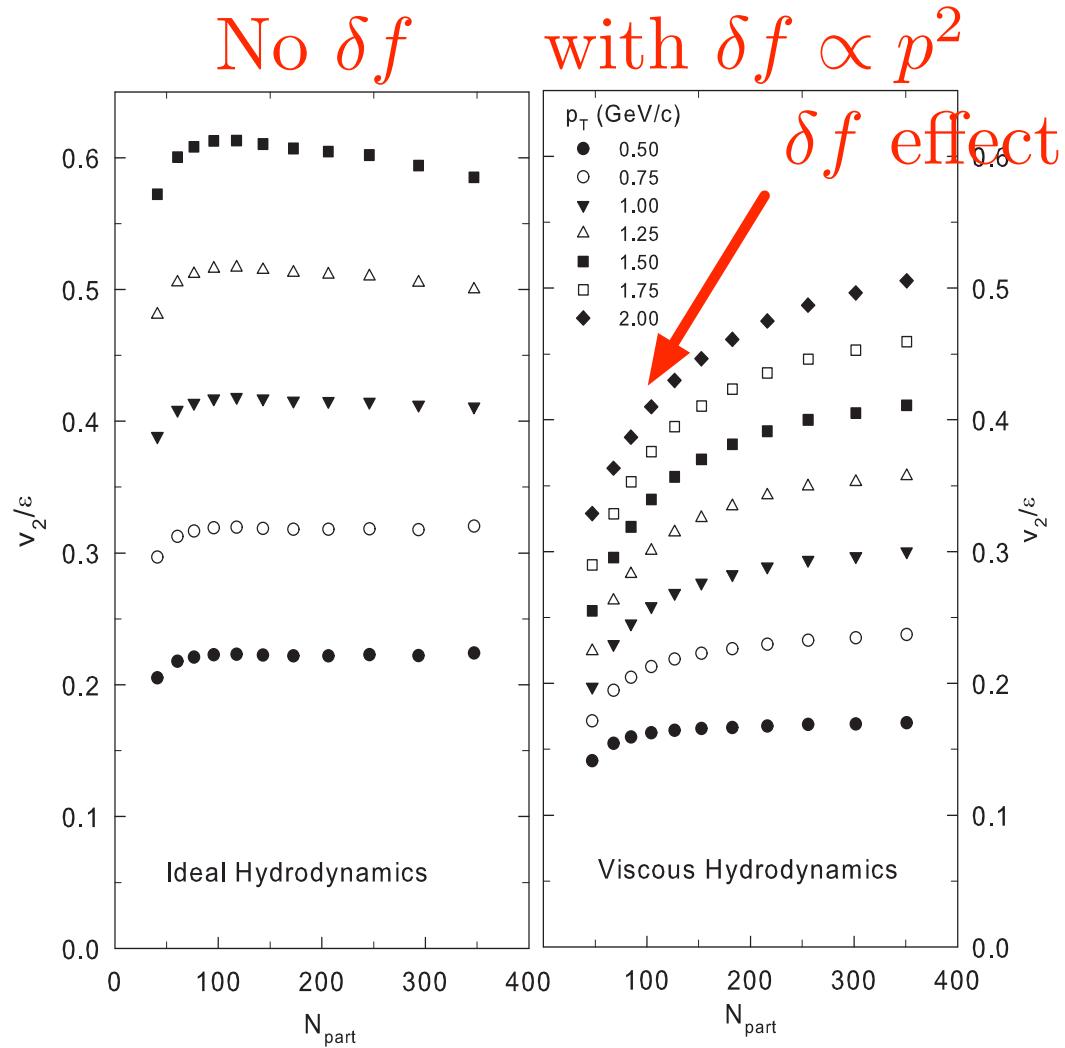


$$\begin{aligned}\hat{q}/T^3 &= 10 \\ \hat{q}/T^3 &= 16 \\ \hat{q}/T^3 &= 60\end{aligned}$$



Determined by $\hat{q}???$

Extracting the $\delta f(\mathbf{p})$ from the data?



Is the p_T and centrality dependence of $v_2(p_T)$ enough?

Multi-component plasmas

Quarks and Gluons

(simple model – see paper for real deal)

- Quarks and Gluons have different relaxation times and δf

$$\begin{aligned}\delta f^Q &= -C_q n_p p^i p^j \langle \partial_i u_j \rangle \\ \delta f^G &= -C_g n_p p^i p^j \langle \partial_i u_j \rangle\end{aligned}$$

- Casimir Scaling

$$\frac{C_q}{C_g} = \frac{\tau_R^Q}{\tau_R^G} = \frac{C_A}{C_F} = \frac{9}{4}$$

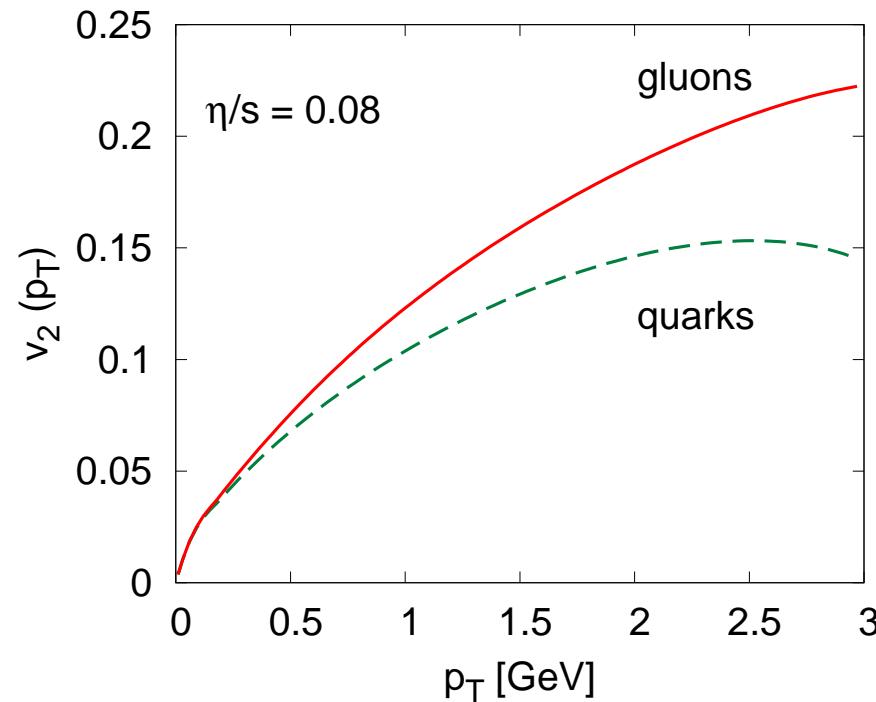
- One constraint is provided by the shear viscosity

$$\eta = \frac{1}{15} \sum_{s=q,g} \nu_s C_s \int \frac{d^3 p}{(2\pi)^3} p^3 n_p (1 \pm n_p).$$

Can now solve for C_q and C_g

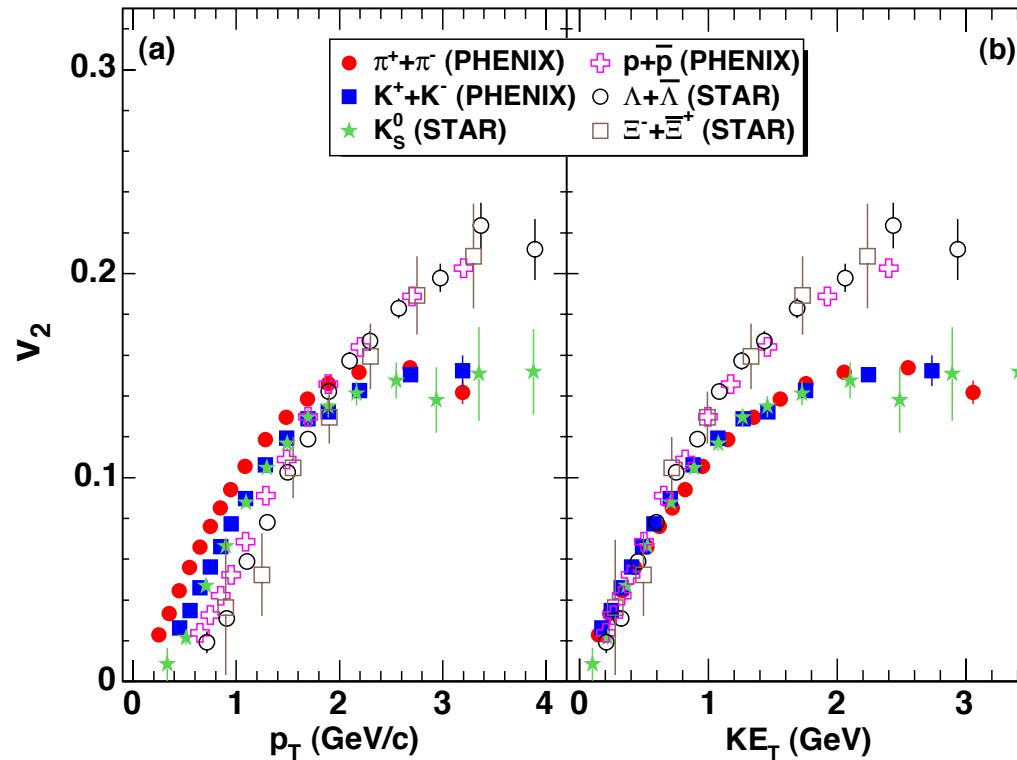
Quarks and Gluons

(simple model—see paper for real deal)



Different Species have different relaxation rates, different δf and different elliptic flows

Mesons and Baryons have different flows



Perhaps they have different relaxation times

Two component meson/baryon gas – relaxation time

$$\delta f_m(p) = -n_p(1+n_p)\chi_m(\tilde{p})\hat{p}^i\hat{p}^j \langle \partial_i u_j \rangle$$

$$\delta f_b(p) = -n_p(1-n_p)\chi_b(\tilde{p})\hat{p}^i\hat{p}^j \langle \partial_i u_j \rangle$$

- Parameterize the viscous corrections as

$$\chi_m(\tilde{p}) = C_m p^2$$

$$\chi_b(\tilde{p}) = C_b p^2$$

- Fit

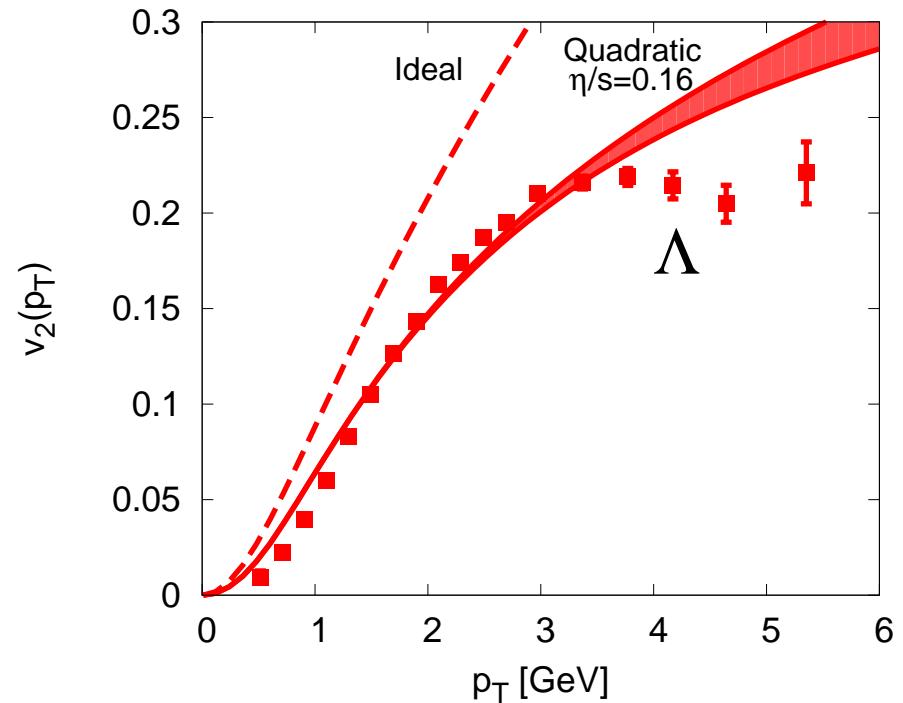
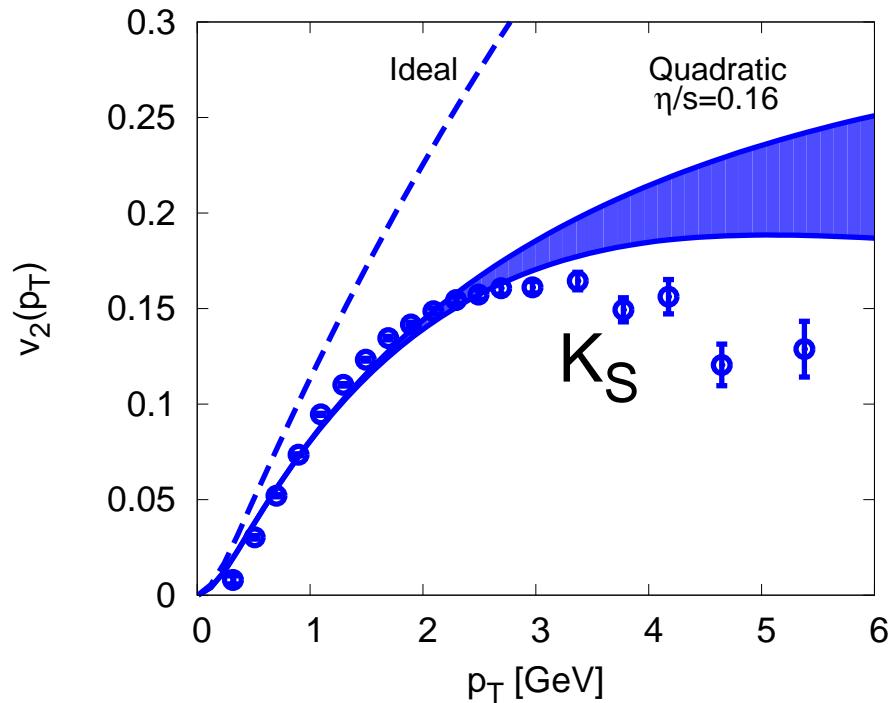
$$\frac{C_m}{C_b} = 1.6$$

- Constrained by shear viscosity

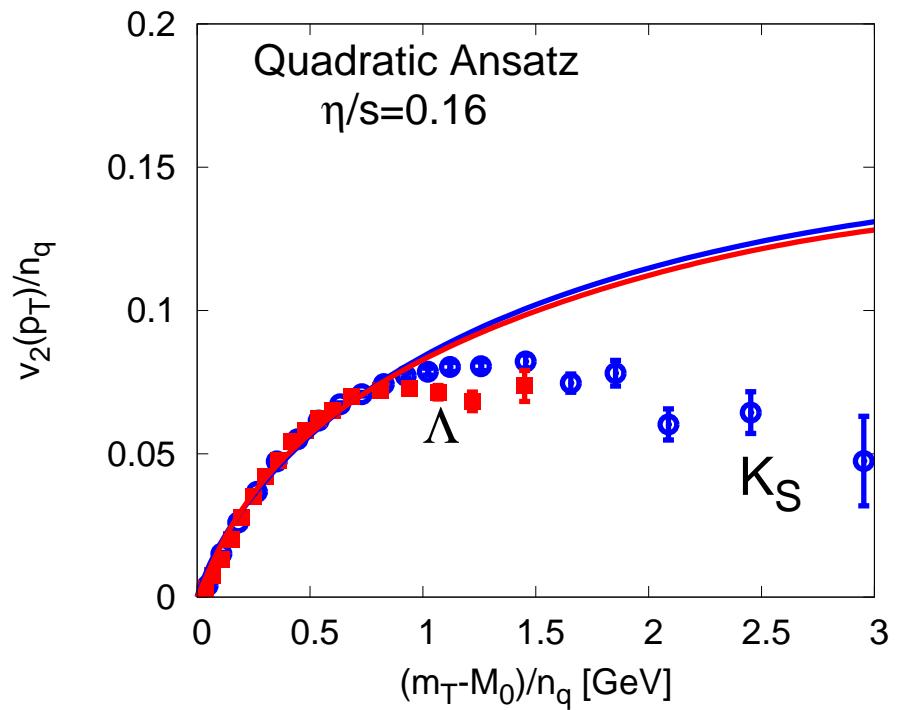
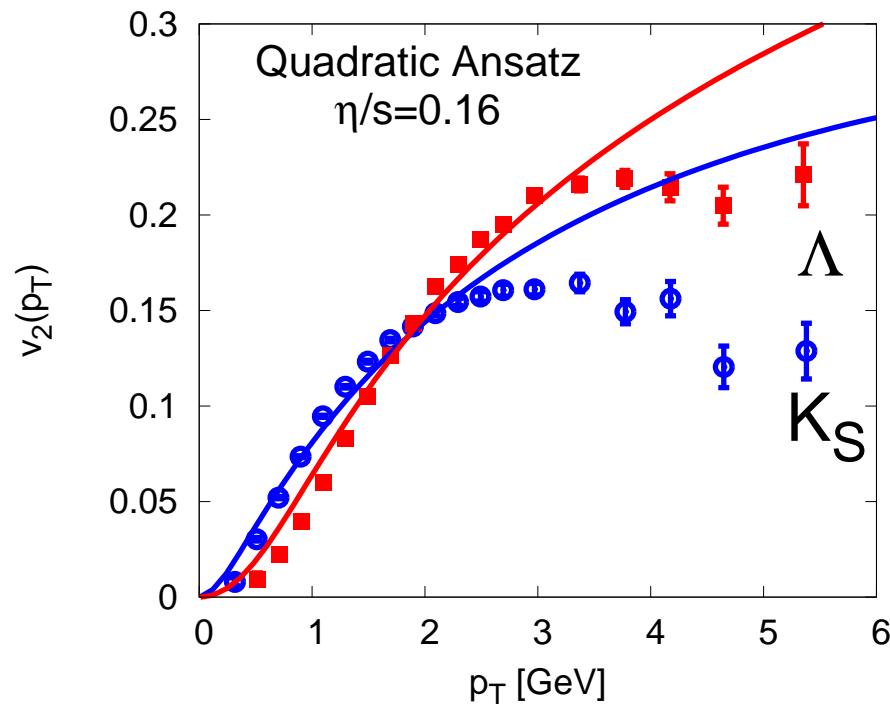
$$\eta = \frac{1}{15} \sum_{a=\pi,K,p,\dots} \nu_a C_{m/b} \int \frac{d^3 p}{(2\pi)^3 E_a} p^4 n(E_a) [1 \pm n(E_a)],$$

No reason to think the relaxation times of baryons are the same as mesons

Results

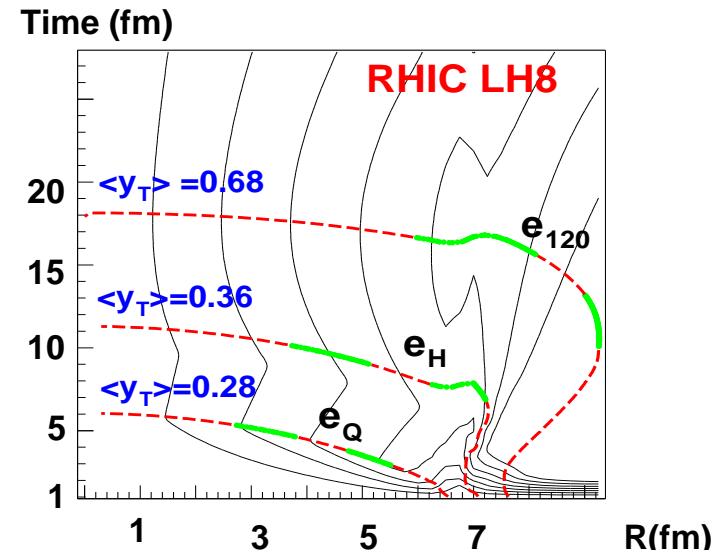
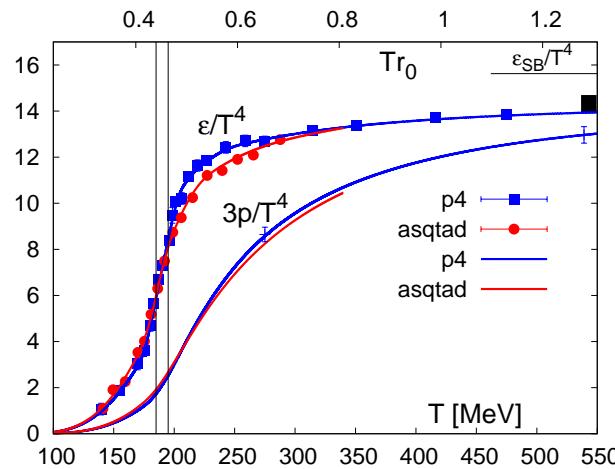


Scaling



Perhaps quark number scaling is simply *Relaxation Time Scaling* (RTS)

Transition Region – long lived, not hadronic or partonic



1. The transition region is long lived ~ 3 fm
2. The interactions are very inelastic in this momentum range

expect $\chi(p) \propto p$

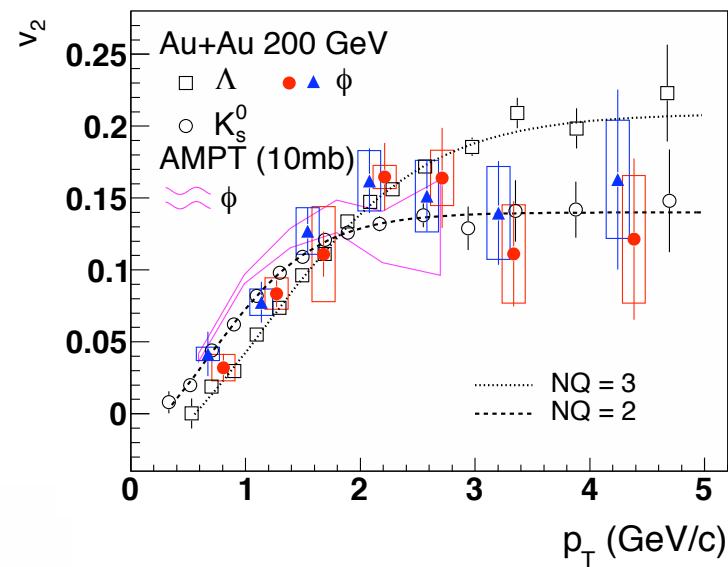
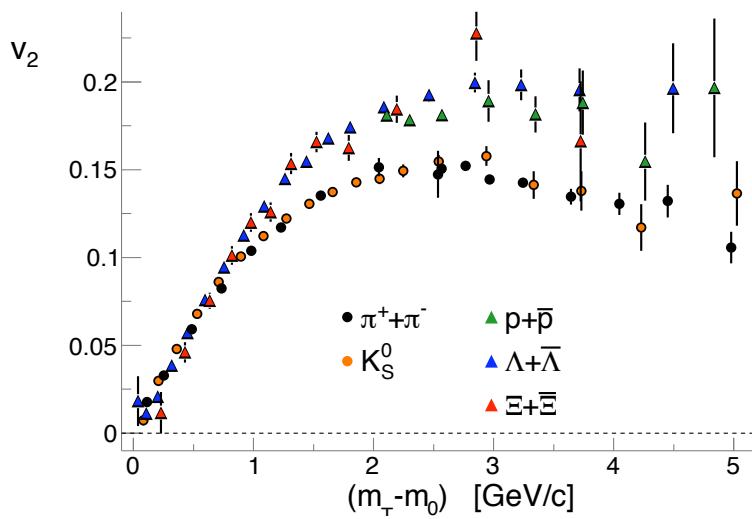
3. Results suggest the geometric additive quark model

(Bleicher et al)

$$\frac{C_m}{C_b} \sim \frac{\sigma_B}{\sigma_M} \sim 1.5$$

Transition Region – approximately $SU(3)$ symmetric

- In the high temperature range expect $SU(3)$ symmetric to be better
 - In $SU(3)$ symmetric world differences Baryon-Meson and spin diffs
 - $\underbrace{\pi, K}_{\tau_{R1}}$ and $\underbrace{p, \Lambda, \Sigma, \Xi}_{\tau_{R2}}$ and $\underbrace{\phi}_{\tau_{R3}}$ and $\underbrace{\Omega^-}_{\tau_{R4}}$



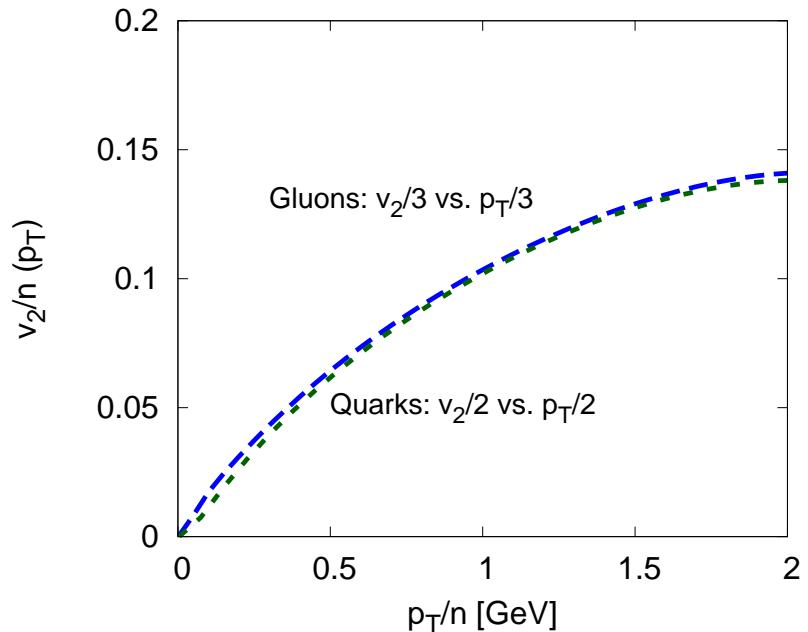
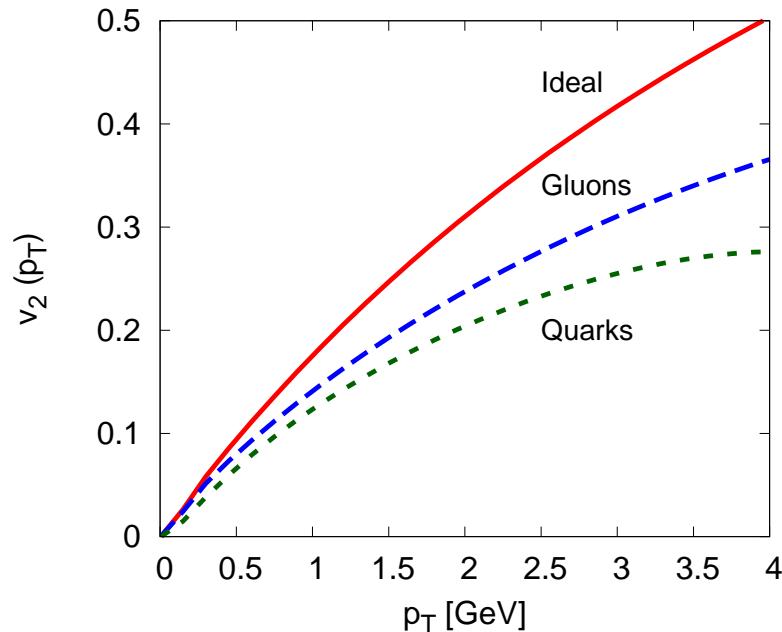
Conclusions

- Studied the kinetics of Quarks and Gluons and the imprints on elliptic flow.
- Radiative energy loss increases the elliptic flow in a certain range

$$p_T \simeq 1.5 \leftrightarrow 2.5 \text{ GeV}$$

- Makes precise the connection between energy loss and viscosity.
- Observed *Relaxation Time Scaling (RTS)* in measured elliptic flow
 - I believe that such relaxation time fits will do as well as coalescence.

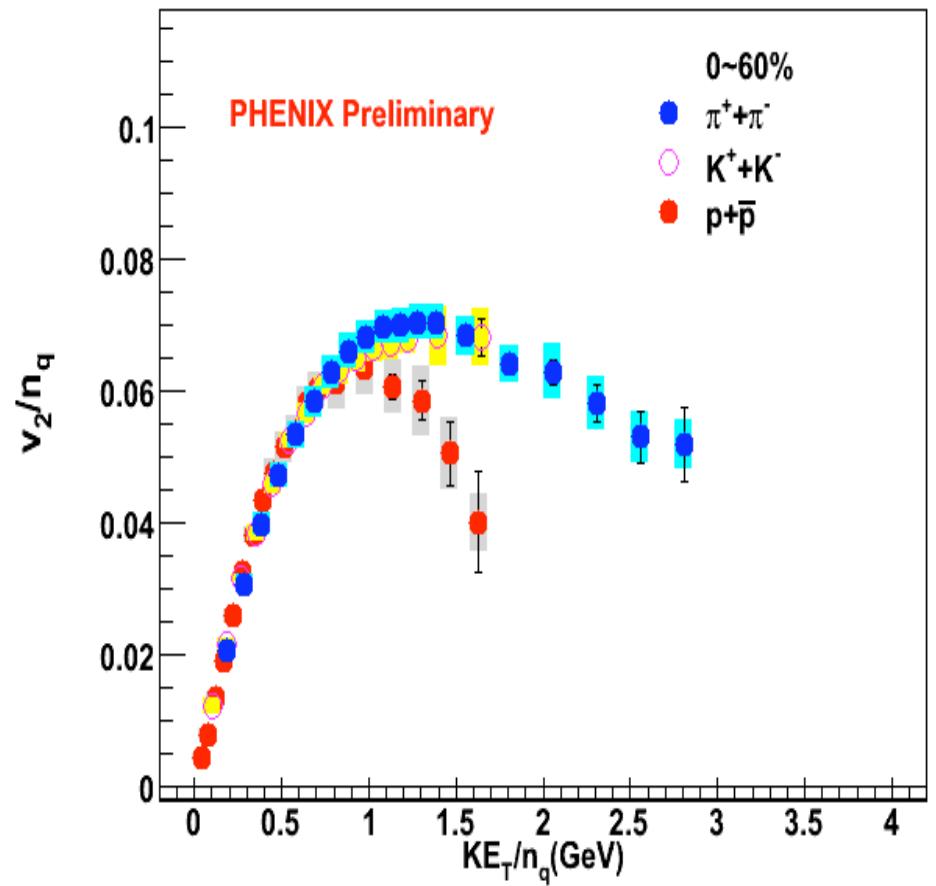
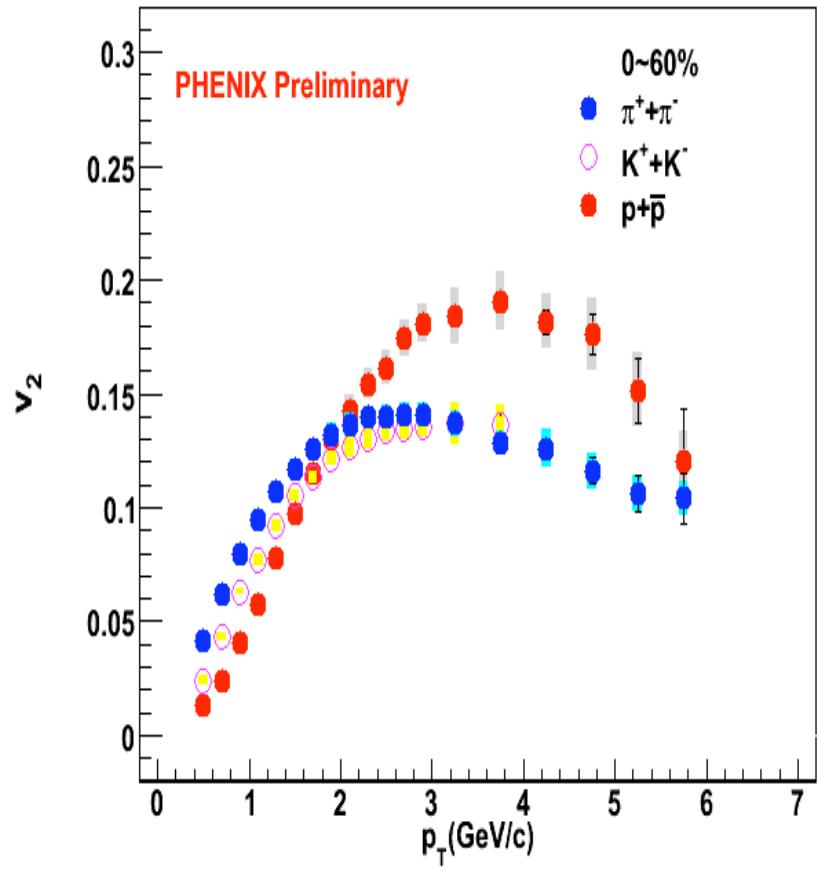
Backup I: perturbative quark and gluon model



“Scaling” can be an artifact of two different relaxation times

Try two different relaxation times for mesons and baryons

Two Components observed: Backup II



Teaney